

Digital Repository of Mathematical Formulae^{*}

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Abstract. The purpose of the NIST Digital Repository of Mathematical Formulae (DRMF) is to create a digital compendium of mathematical formulae for orthogonal polynomials and special functions (OPSF) and of associated mathematical data. The DRMF addresses needs of working mathematicians, physicists and engineers: providing a platform for publication and interaction with OPSF formulae on the web. Using MediaWiki extensions and other existing technology (such as software and macro collections developed for the NIST Digital Library of Mathematical Functions), the DRMF acts as an interactive web domain for OPSF formulae. Whereas Wikipedia and other web authoring tools manifest notions or descriptions as first class objects, the DRMF does that with mathematical formulae. See http://gw32.iu.xsede.org/index.php/Main_Page.

1 Introduction

Compendia of mathematical formulae have a long and rich history. Many scientists have developed such repositories as books and these have been extremely useful to scientists, mathematicians and engineers over the last several centuries (see [2, 3, 5, 6, 9, 13, 15] for instance). While there may be some overlap of formulae in different compendia, one often needs to be familiar with many different compendia to find a specific desired formula. Online compendia of mathematical formulae exist, such as the NIST Digital Library of Mathematical Functions (DLMF), subsets of Wikipedia, and the Wolfram Functions Site. We hope to take advantage of the best aspects of these online efforts while also incorporating powerful new features that a community-arm of scientists should find beneficial. Our strategy is to start with validated and trustworthy special function data from the NIST DLMF, while adding Web 2.0 capabilities which will

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encourage community members to discuss mathematical data associated with formulae. These discussions will include internally hyperlinked proofs as well as mathematical connections between formulae in the repository.

The online repository will be designed for a mathematically literate audience and should (1) facilitate interaction among a community of mathematicians and scientists interested in formulae data related to orthogonal polynomials and special functions (OPSF); (2) be expandable, allowing the input of new formulae; (3) be accessible as a standalone resource; (4) have a user friendly, consistent, and hyperlinkable viewpoint and authoring perspective; and (5) contain easily searchable mathematics and take advantage of modern MathML tools for easy to read, scalably rendered mathematics. It is the desire of our group to build a tool that brings the above features together in a public website for mathematicians, scientists and engineers. We refer to this web tool as the Digital Repository of Mathematical Formulae (DRMF).

Our project was motivated by the goal of creating an interactive online compendium of mathematical formulae. This need was addressed in SIAM Activity Group OPSF-Net discussions, such as Dmitry Karp (OPSF-Net 18.4, Topic #5). In that OPSF-Net edition, there were two related posts (OPSF-Net 18.4, Topics #6,#7) with a follow-up post in OPSF-Net 18.6, Topic #3.

2 Implementation

In our project, we have taken advantage of the free and open source MediaWiki wiki software as well as tools developed within the DLMF project [14], such as \LaTeX XML and the DLMF \LaTeX macros. DLMF macros (and extensions as necessary) tie specific character sequences to unique mathematical objects such as special functions, orthogonal polynomials, or to other mathematical symbols associated with these. The DLMF macros are hence used to define OPSF within DRMF and through \LaTeX XML, their corresponding rendered mathematical symbols. Furthermore, the use of DLMF macros as linked to their definitions within the DLMF, allows for easy access to precise OPSF definitions for the symbols used within the \LaTeX source for OPSF formulae. The committed use of DLMF macros guarantees a mathematical and structural consistency throughout the DRMF. As a web tool, the DRMF provides (1) formula interactivity, (2) formula home pages, (3) centralized bibliography, and (4) mathematical search. The DRMF shares the core DLMF component, \LaTeX XML, which (through the MediaWiki Math extension) processes Wikitext math markup written in \LaTeX to produce XML and HTML. For formula interactivity and menus linked to formulae, we have utilized the JOBAD interactivity framework and are investigating the use of MathJax [4]. We have also incorporated the MediaWiki: Math and MathSearch [16] extensions. Within the DRMF, we will develop technology for users to interact with formulae using a clipboard, which allows for easy copy/paste of formula source representations (to include \LaTeX with DLMF macros; presentation or content MathML; as well as input formats for computer algebra systems such as Mathematica, Maple, and Sage).

The DRMF treats formulae as first class objects, describing them in formula home pages that currently contain: (1) a rendered description of the formula itself (required); (2) bibliographic citation (required); (3) open section for proofs (required); (4) list of symbols used and links to their definitions corresponding to the DLMF macros (required); (5) open section for notes relevant to the formula (e.g., formula name, if the formula is a generalization or specialization of some other formula, growth or decay conditions, links to errata pages, etc.); (6) open section for external links; (7) substitutions with definitions required to understand the formula; and (8) constraints the formula must obey. For each formula home page there is a corresponding talk page, and we are incorporating a strategy for handling the insertion of formula errata. A major resource in our ability to implement effective and precise OPSF search will be the use of the DLMF macros in building the \LaTeX source for OPSF formulae and related mathematical data.

Next, we present an overview of the seed resources, which we plan to incorporate within DRMF. We have been given permission and are seeding the DRMF with data from the NIST DLMF [14]. We have also been given permission to and are seeding \LaTeX formulae data from [11] (KLS). We will also incorporate Tom Koornwinder's companion of recent arXiv published additions to KLS [12]. We have also been given permission to incorporate seed formula data from [5, 6] (BMP). Efforts to upload BMP data, as well as any book data without existing \LaTeX source, will prove extremely difficult, since this effort will rely on the use of mathematical optical character recognition (OCR) software such as InftyReader to produce \LaTeX source for these formulae. Mathematical OCR is still in its nascence and this effort is currently under consideration for feasibility of use. We are in communication with other authors and publishers to gain access and permission for other proven sources of mathematical OPSF formulae such as [1, 7, 9, 10] and we are excited about the prospect of seeding proof data by Victor Moll and collaborators (see for instance [8]). For \LaTeX source where DLMF macros are not present (such as KLS), we are developing tools which automate DLMF macro replacements. Seeding and generating symbol lists are accomplished by converting \LaTeX source into Wikitext, in an automated fashion. We use `Pywikibot` to automate the uploading of Wikitext pages to our demo site.

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