



Preface

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Published online: 24 April 2023

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These memorial issues in the Ramanujan Journal represent a modest contribution to commemorate the great man, Richard (Dick) Askey. Let us give a little context here so that it is appreciated how deep an influence he had and why it was that so many individuals participated. When Dick started his career, there was a widespread belief that numerical and computational techniques would replace the use of special functions in their applications to physical sciences and engineering. There were those who made detailed attempts at presenting the state of the art and advancing the field of special functions, such as Yudell Luke, Bille Carlson, Felix Arcsott, and Ian Sneddon. A few years earlier, the Bateman Manuscript Project [1–3] was a major effort at curation and compilation of the mathematical theory of special functions. However, at the same time, one of its major proponents, Arthur Erdélyi, after the completion of his monumental efforts, moved to generalized functions, operational calculus and differential equations. Furthermore, many of the papers published in special functions, at the time, covered what Dick would call “curiosities”. Others, such as Leonard Carlitz, made important contributions to the subject but it seems that his contributions were barely noticed. The subject seemed to be in a state of lethargy. Then Dick Askey appeared on the scene. As a harmonic analyst, Dick needed properties of certain orthogonal polynomials and special functions, especially inequalities. To his surprise much of what he wanted was not in the literature, so he gradually started developing new properties of old functions and he attracted students and younger mathematicians. Mourad first met Dick around 1970, and like many others from his generation, was very impressed by Dick’s encyclopedic knowledge of mathematics, his love of special functions, and his charming lecturing style.

When the SIAM Journal on Mathematical Analysis (SIMA) started in 1969, Dick joined the editorial board. For the next twenty years or so, this journal published many of the most important papers in the field. The journal also had other editors who were leaders in the field such as Willard Miller, Jr. and Walter Gautschi. The first

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managing editor Frank W. J. Olver, who started the journal, worked on the asymptotics of special functions. His contributions to Abramowitz and Stegun (National Bureau of Standards Applied Mathematics Series · 55) [4] were top tier, just as his later efforts as editor-in-chief of the NIST Digital Library of Mathematical Functions (DLMF) project [5], in which Askey was co-author for three DLMF chapters and member of the original editorial committee. Dick read most of the papers he handled at SIMA, and in many cases he suggested concrete ideas for improving the results in the paper and/or suggestions for future work. Dick was super generous with ideas and had an amazing intuition.

Askey's contributions to mathematics are far and great, and we encourage the reader to look at a recent detailed survey article on Dick's contributions during the course of his career [6]. In our opinion, Dick's most influential work in mathematics was done during the twenty-year period from 1970 to 1990 (after 1990, Dick devoted most of his time and energy to mathematics education).

Dick's work with George Gasper on inequalities played a crucial part in the proof of the Bieberbach conjecture by Louis de Branges. He supervised Jim Wilson's thesis where the Wilson polynomials were developed. Here Dick's hints and suggestions played a central role and were clearly acknowledged [7]: "For creating the atmosphere in which all the ideas in this thesis sprang up inevitably, my gratitude goes to Professor Richard Askey." Their q -analogue, the Askey–Wilson polynomials were introduced in the memoir [8]. These orthogonal polynomials occupy a central place in Askey's contributions and are important within many mathematical disciplines. Askey also suggested to Mourad to look at the combinatorics of integrals of products of the type

$$\int_{\mathbb{R}} \prod_{j=1}^k p_{n_j}(x) f(x) d\mu(x), \quad (0.1)$$

where $\{p_n(x)\}$ is an orthogonal polynomial system with respect to the measure $d\mu$. This led Dominique Foata, Xavier Viennot, and several others to develop a beautiful combinatorial theory of orthogonal polynomials. Dick also supervised Dennis Stanton's thesis which was on combinatorics and discrete mathematics which was connected to special functions. Dick also realized the importance of the Selberg integral and its q -analogues and pointed out many ingenious interpretations of the formulas of Ramanujan. We would say that Ramanujan's work had a major lift throughout many different areas of mathematics. Dick was a huge proponent of the work and influence of Ramanujan. Dick Askey even moved Ramanujan's wife with his efforts in the commissioning of at least four bronze busts of Ramanujan made by the famous artist Paul Granlund.

The papers in this special issue are written by many of Dick's friends, from different areas, including analysis, combinatorics, and number theory. Let us start with the papers which build on Askey's work. The Arvesu–Driver–Littlejohn paper "Zeros of Jacobi and ultraspherical polynomials" proves and extends a conjecture of Dick Askey about the interlacing of the zeros of the Jacobi polynomials $P_n^{(\alpha, \beta)}(x)$ and $P_n^{(\alpha, \beta+2)}(x)$. Their dedication reminded us that Dick's favorite special functions were Jacobi polynomials. The methods of proof in the Bhatnagar–Ismail paper originated in

the Askey–Ismail memoir published in 1984 [9]. The Christiansen–Simon–Zinchenko paper deals with asymptotics of Chebyshev polynomials and is related to the general theory of orthogonal polynomials. Although the paper is not directly based on Askey’s work, Askey’s detailed editorial comments on the collected works of Gábor Szegő [10–12] were very helpful in motivating Simon to start his work on orthogonal polynomials. We would also put the Cohl–Costas–Santos article in this category in connection with Askey’s focus on the importance of q -beta integrals. The paper by Gasper gives an upper bound for a quotient of products of hypergeometric functions. The author says “The above conjecture turned out to be the last open problem that Dick encouraged me to solve.” Gasper not only solved the problem but also established a more general result. The problem originated in the financing efficiency of securities-based crowdfunding. Askey had a special interest in addition theorems. In fact in his Foreword to Miller (1977) [13] he says, in regard to the addition theorem for spherical harmonics and the corresponding addition theorem for trigonometric functions, “[these] are among the most important facts known about these functions.” Not only that, but Askey was also especially interested in the continuous q -ultraspherical/Rogers polynomials and continuous q -Hermite polynomials; so the Koornwinder paper certainly goes a long way towards Askey’s direct interests. Askey’s original intuition that computing moments would be hard was confirmed by the realization that the determination of moments is an interesting combinatorial problem. The papers by Krattenthaler and Gessel–Zeng would therefore have been of interest to him. Askey was also very interested in the Rogers–Ramanujan identities as well as q -calculus, q -series and properties of basic hypergeometric functions, so the papers by Rosengren (which includes identities which are very similar to the Rogers–Ramanujan identities), Kim–Stanton, Wang–Yee, Kalmykov–Karp–Kuznetsov and Terwilliger would have been very much appreciated by him. The Forrester–Kumar paper deals with Selberg integrals which, as mentioned previously, attracted Dick’s attention.

The next group of papers in these Richard Askey Memorial Issues lie in areas indirectly related to Dick’s work. The great collaboration between George Andrews (whose background was in number theory, but has great results in analysis and combinatorics as well), Dick Askey, and also Bruce Berndt, spawned huge advances in many fields including the study of Ramanujan’s work. These memorial issues also include some technical number theoretic results including the papers of Alladi, Andrews–Uncu, Andrews–Dragovic–Radnovic, Baker, Bringmann–Ono–Wagner, Bringmann–Jennings-Shaffer–Mahlburg, Elliott–Kish and Garvan. The work of Dai–Yao deals with random matrix theory, Bessel functions, and multiple and discrete orthogonal polynomials. The Groenevelt–Koelink paper studies a Lax pair in a 2-parameter Lie algebra in various representations. Although Askey never used Lax pairs, he promoted the study of special functions and polynomials through representation theory and harmonic analysis. The Diaconis–Zhong paper uses Hahn polynomials and as the authors say “Richard Askey’s support and encouragement was crucial throughout our fledgling efforts to learn and apply the beautiful subject that he built.”

Askey recognized the power of symbolic algebra since its infancy and was very supportive of the pioneering work of Ralph William (Bill) Gosper. Nowadays we all use symbolic algebra packages and some would even wonder what life was like before these powerful packages and computers. The Dougherty–Bliss–Zeilberger paper gives

a MAPLE package and a doubly infinite family of continued fractions. The authors remind us in their dedication that Askey taught us that “special functions are useful functions.” Dunkl’s article is on a superpolynomial version of nonsymmetric Jack polynomials. Dunkl’s dedication is to the memory of Dick Askey, “who was my special functions teacher, and who made it respectable to find exact answers to analysis problems.” In the Schlosser–Zhou paper, they remark that in September 2019, as a result of experimentation using computer algebra, Schlosser presented a paper containing a conjecture in a tribute dedicated to Richard Askey [14, 15]. One of the main results of the Schlosser paper is a partial affirmation of this conjecture (Corollary 5). Askey was also interested in Sylvester-type determinants [16], so the Johnson paper would have been interesting to him. Askey was also interested in biorthogonal functions (see for example [17]), and also the Askey scheme (originally drawn and distributed by Jacques Labelle [18]), so certainly the Tsujimoto–Vinet–Zhedanov paper would be in his interests. In their dedication, they refer to Askey as “mathematician and pedagogue extraordinaire.” Askey was also interested in theta functions and bilateral series, so the paper by Liu should have attracted his attention.

After a long reviewing period, the end product, these Richard Askey Memorial Issues in the Ramanujan Journal, contain 31 papers submitted by 56 authors. Richard Askey was a Founding Editor of The Ramanujan Journal. These papers in memory of Askey are being published in two consecutive special issues, this being the first of two. The articles in each memorial issue are arranged chronologically as per the date of submission, as is the practice with every Ramanujan Journal issue.

Askey had a great influence on the field of special functions and on the individuals in that field. So many of us owe so much to Dick, and it is the hope that these Richard Askey Memorial Issues will go a short way to expressing our admiration for the legacy he has left us. We conclude by quoting Luc Vinet’s words from the *Liber Amicorum* [14], which was presented to Dick before he passed away. “Dick, many many thanks for your mathematics and for encouraging us all to climb on the shoulders of the giant that you are.”

Acknowledgements Fond remembrances should go to Mizan Rahman (1932–2015) and Ranjan Roy (1948–2020), both of whom we dearly miss and would have loved to participate in these Richard Askey Memorial Issues. Thanks also to Loyal Durand whose fond recollections of Dick Askey inspired Howard during the writing of this preface, and to Michael Schlosser for valuable discussions.

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